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Reply by Author to L. W. Schwartz

J. C. Hill*
Oakland University, Rochester, Mich.

SCHWARTZ raises two criticisms of my recent Technical Note.[1] One is, I think, minor; the other major. First, the minor one; presentation of results at Mach 1 was not intended to imply that Eq. (1) of my Note was valid in the transonic flight regime: it was intended merely to show how well the expression approximated the data in the transonic regime, leaving to the user the technical judgment, dependent on his particular aerodynamic configuration and the accuracy he requires, of how far into the transonic region he may go and still assume that the data of Fig. 1 of my Note represents reality.

Schwartz's major criticism that more exact expressions representing the same data exist is, to my mind, more to the point—and he shows quite clearly that there are such expressions. As I meant to make clear, my expression was developed from Nielsen's[2] Fig. 9–6, *not* from the data from which Fig. 9–6 was prepared (which is, apparently, either Ref. 3 or 4; Nielsen does not state which).† The accuracy (or lack thereof) of the present expression is, I believe, primarily because of the graphical nature of the data from which it was developed and because of the limited range of Mach number covered. No attempt was made to improve its accuracy because it was adequate to the purpose for which it was needed—which was to provide order-of-magnitude estimates of drag coefficients for use in *control* studies, where 5% accuracy would be quite adequate, as opposed to performance studies, where much better accuracy would be desirable. The expressions pointed out by Schwartz apparently provide significantly greater accuracy than my expression.

I am indebted to Mr. Schwartz for bringing these more accurate expressions to my attention. The utility of such expressions for estimating aerodynamic characteristics of complex configurations seems obvious, and a variety of standard expressions for lift, drag, etc. must be in common use by those who deal with such problems on a day-to-day basis.

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* Associate Professor of Engineering.

† Schwartz treats the Sims data as exact; therefore, Schwartz's Lockheed expression and Nielsen's Fig. 9-6 were obtained from different sources. I have not yet been able to compare the two sources; one would hope that this would not be a significant source of error.

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Comment on "Drag Coefficient of Small Spherical Particles"

Frederick L. Schuyler*
General Electric Company, King of Prussia, Pa.

THE authors[1] have presented one of the latest experimental studies of small particle acceleration. Additional work, presented in Ref. 2 but not widely distributed, might also be of interest to the authors.

Schuyler carried out both an analytical and experimental study of the trajectory of small (750–1250 μ diam) evaporating liquid droplets. Freon 12 at $-22°F$ was used. Freon droplets were injected into a 12-in. test chamber in which 86°F air moved with a velocity that increased linearly from entrance (50 fps) to exit (200f ps). 6000 frame/sec movie pictures were taken.

Freon 12 was used primarily because of its high evaporation rate at the test conditions. The study was aimed at developing a basic understanding of the dynamics of spray vaporization in the combustion chamber of liquid propellant rockets. The main purpose of the experiments was to check the validity of two major assumptions in the theoretical analysis. These were: the drag coefficient is given by Stoke's law $C_D = 24/Re$, and the droplets' diameter-time variation may be represented by $d^2 = d_0^2 - \lambda t$. Here, d is diameter, d_0 initial diameter, and λ evaporation rate coefficient. Of interest now is the value of C_D, although it should be mentioned that the $D^2 - \lambda t$ law was valid only up to a droplet velocity of 300 fps. Above that, other phenomena take place.

As in the author's study, droplet diameter d, acceleration α, and relative velocity U_R were required to be determined from the experiments. Droplet density ρ' as well as atmospheric density ρ and viscosity μ were assumed constant at $-22°$ and 86°F, respectively. The drag coefficient was calculated as follows:

$$C_D = F_D/\tfrac{1}{2}\rho A U_R^2$$

$$F_D = \rho'\alpha(\pi d^3/6), \quad A = \pi d^2/4$$

$$\alpha = \Delta V/\Delta t = (V/\Delta x)\Delta V, \quad Re = \rho U_R d/\mu$$

Therefore,

$$C_D = \{[\tfrac{4}{3}(\rho'/\mu)]d^2\alpha/U_R\}/Re$$

Measured during the experiments were, droplet position and time (relative to a length scale placed in the test chamber) and local test chamber pressure (from which local air velocity was calculated). Droplet diameter was determined from the photographic records. Each frame of interest was magnified and the droplet measured in four directions: horizontal, vertical, and two diagonals. To determine droplet velocity and acceleration, a curve was passed through the droplet's position-time data. This curve was then differentiated using local finite differences to obtain velocity and acceleration. Though individual large errors probably occurred in this manner, it is expected that the number of data points were sufficiently large to yield reasonably correct values of droplet velocity and acceleration. The results of these experiments

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*Consulting Scientist, Space Sciences Laboratory. Member AIAA.

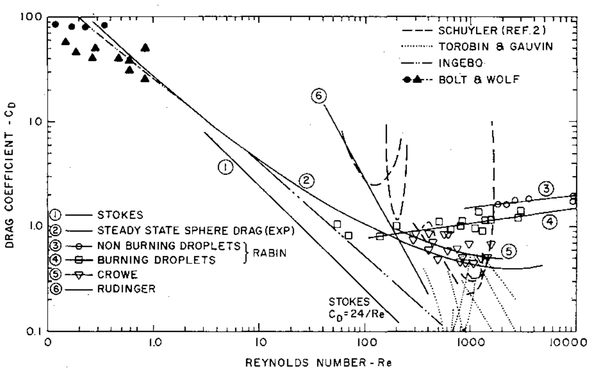

Fig. 1 Small particle drag coefficient vs Reynolds number.

are superimposed on Fig. 2 of the author's paper and presented here as Fig. 1. Also shown is Stoke's law.

The range of Reynolds number studied was 500–1500. Each of the curves of the Schuyler data in Fig. 1 represents the history of a single droplet. Each curve starts at a low *Re* and goes toward a large *Re*. This is due to the air (and also relative) velocity increasing faster than the decrease of the droplet's diameter. It is remembered that the air velocity was designed to increase linearly with distance.

Perhaps the largest droplet measurement error was that associated with its diameter. Initially the droplets are relatively spherical, but as they proceed down the chamber (Fig. 2), the dynamic pressure force flattens them, creating a large area perpendicular to the air flow. However, in the photographs only their side area could be seen. This means that the actual drag coefficient should be larger than that recorded in Fig. 1. This is particularly true for the higher Reynolds number (larger diameter) droplets. The smaller droplets did not appear to flatten as much. In Fig. 2 can be seen a relatively spherical droplet, a highly flattened droplet, and a droplet which has been distended into a cup shape whose stretched membrane is just about to rupture.

The present data, without correction for droplet diameter, correlate approximately with Rudinger's glass bead results. If, however, a more appropriate diameter (associated with the frontal area) could be measured, the present high *Re* data would be corrected probably upward and most likely would fall more in line with Rabin's data.

Reported in Ref. 2 is the possibility that for a given Reynolds number, the drag coefficient decreases with acceleration. The data of Ref. 2 may be put in three acceleration groups. For $Re \approx 150$, $\alpha \approx 80$ ft/sec^2; $Re \approx 400$, $\alpha \approx 32$ ft/sec^2; and $Re \approx 1000$, $\alpha \approx 200$ ft/sec^2. The data of Ingebo in Fig. 2 have an acceleration of approximately 10,000 ft/sec^2. Calculating the acceleration modulus for Schuyler's data yields $Ac \approx 10^{-4}$. According to Crowe,[3] this should indicate that the droplet's drag coefficient should be negligibly effected by acceleration. For the present drag data, however, this does not appear to be the case. If the data were corrected for frontal area, however, acceleration effects might drop out.

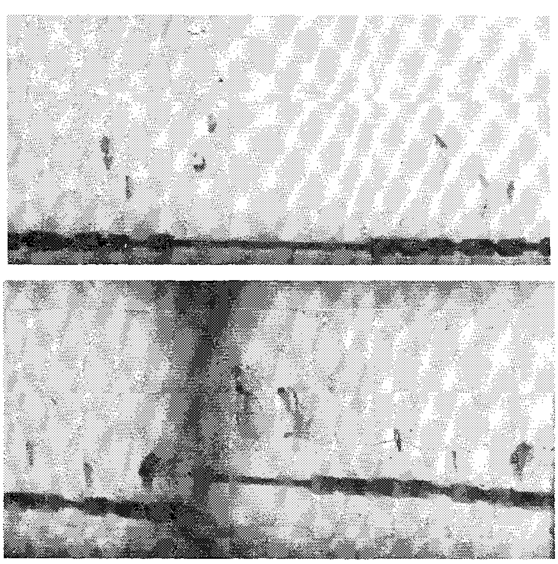

Fig. 2 Photograph of liquid droplet (Freon –12, CCl_2F_2) deformation in a linearly increasing air velocity field (two successive frames, the first one on top).

The relative Mach number M_R for the data of Ref. 2 was approximately 0.05. Therefore, the relative Mach number effect referred to in the authors paper should be of no concern in the interpretation of Schuyler's data. Because higher values of M_R were not studied by Schuyler, no corroboration of the Mach number effect is possible.

In conclusion, it can be stated that the authors contention that small particle drag coefficients can be expected to be substantially higher for rocket combustion chamber conditions than quiescent conditions, appears to be further substantiated by the data of Ref. 2. Though proper reduction of the higher *Re* data of Ref. 2 was hampered by the inability to measure droplet frontal area, such a correction to the data would yield even higher values of drag coefficient.

It is agreed that even in light of past investigations, a substantial amount of work still is required to determine the effects of compressibility, of low Reynolds number, burning (mass loss), neighboring particles, and turbulence on small particle trajectories. For deformable particles, photographic data on frontal area variations must be obtained also.

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Reply by Author to F. L. Schuyler

J. A. NICHOLLS*
University of Michigan, Ann Arbor, Mich.

IN the Comment immediately preceding this, Schuyler refers to the work of Selberg and Nicholls[1] and presents some of his experimental results on the drag and acceleration of evaporating liquid drops. On the basis of these results, Schuyler poses the possibility that acceleration effects are important in his drag data. I seriously doubt this, for, as he has indicated, there are other effects involved in the acceleration of the liquid drops. If one calculates the range of Weber and Reynolds numbers for the experiments, it is apparent that the drops must undergo a transition from a bag to a stripping mode of disintegration. Ranger and Nicholls[2] have presented results for the stripping mode that indicate that the drop diameter increases to more than 3 times the initial diameter in the transverse direction. The windward face takes on an elliptical profile and the leeward face, while usually not visible, probably takes on a concave shape. At any rate, the drop distorts to a shape far from that of a

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*Professor, Aerospace Engineering. Associate Fellow AIAA.